Kwantowa teoria gier w podejmowaniu decyzji

Marek Szopa

Department of Operations Research

marek.szopa@uekat.pl

University of Economics in Katowice

Opinions on quantum mechanics

Richard Feynman (1918-1988)

I think it is safe to say that no one understands quantum mechanics. Do not keep saying to yourself, if you can possibly avoid it, "But how can it be like that?" because you will get "down the drain" into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that.

- Richard Feynman

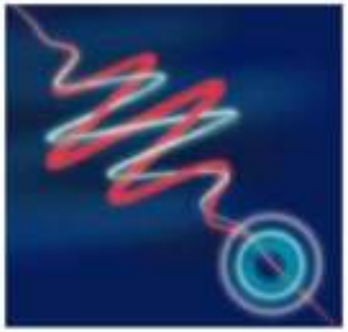
Those who are not shocked when they first come across quantum mechanics cannot possibly have understood it.

- Niels Bohr

Our plan

- Quantum computing
- Game theory in economics
- EWL approach to quantum game theory
- Pareto efficiency of quantum mixed equilibria
- Quantum absentminded driver
- IBM Q simulations

Quantum computing advantages

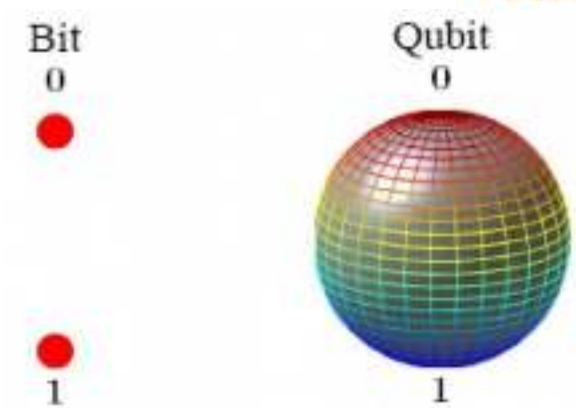


Wave-particle duality → every particle or quantum entity may be described as either a particle or a wave..

- It expresses the inability of the classical concepts "particle" or "wave" to fully describe the behavior of quantum-scale objects.
- IT IS USED TO INTERACT WITH QUBITS THROUGH INTERFERENCES.

Probabilistic system → any given state can be observed.

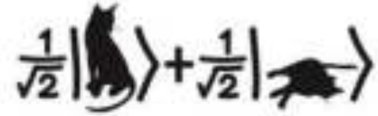
- There is a computable probability corresponding to the likelihood that any given state will be observed if the system is measured.
- Quantum computation is performed by increasing the probability of observing the correct state to a sufficiently high value so that the correct answer may be found with a reasonable amount of certainty.
- A QUANTUM RESULT IS GENERALLY AN EVALUATION OF THE QUBITS FINAL STATES.



In general, a quantum computer with n qubits can be in any superposition (as Schrodinger's cat) of up to 2^n different states. This compares to a normal computer that can only be in one of these 2^n states at any one time.

Quantum Technologies. Market and Technology Report 2020. Yole Developement

Quantum computing advantages



Superposed states → can be in all possible states at the same time.

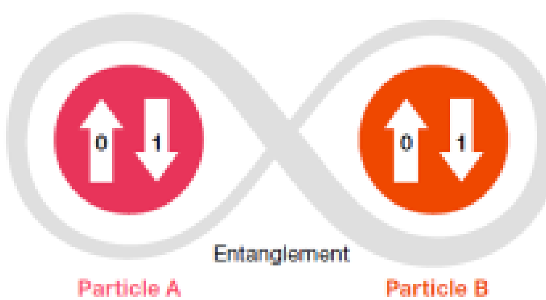
- With respect to a quantum computer, this means that a quantum register exists in a superposition of all its possible configurations of 0's and 1's at the same time, unlike a classical system whose register contains only one value at any given time. It is not until the system is observed that it collapses into an observable, definite classical state. For example, the electron spin can be up and down at the same time.
- **THIS ALLOWS SUPERPOSED CALCULATIONS, THUS DRAMATICALLY DECREASING COMPUTING TIME**

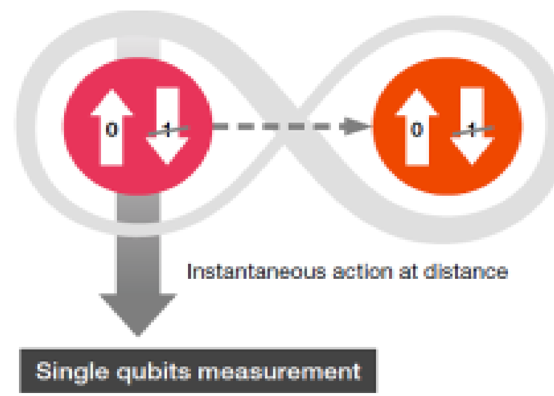
Quantum Computing. A technology of the future already present. PwC point of view 2019

Entanglement → cannot be decomposed into more fundamental part.

- Two distinct elements of a system are entangled if one part cannot be described without taking the other part into consideration.
- An especially interesting quality of quantum entanglement is that elements of a quantum system may be entangled even when they are separated by considerable space.
- Quantum teleportation, an important concept in the field of quantum cryptography, relies on entangled quantum states to send quantum information adequately accurately and over relatively long distances.
- **ENTANGLEMNT IS USED TO LINK THE QUBITS (2 or 3-qubits logic gate) IN QUANTUM COMPUTING AND SYNCHRONIZE THEM.**

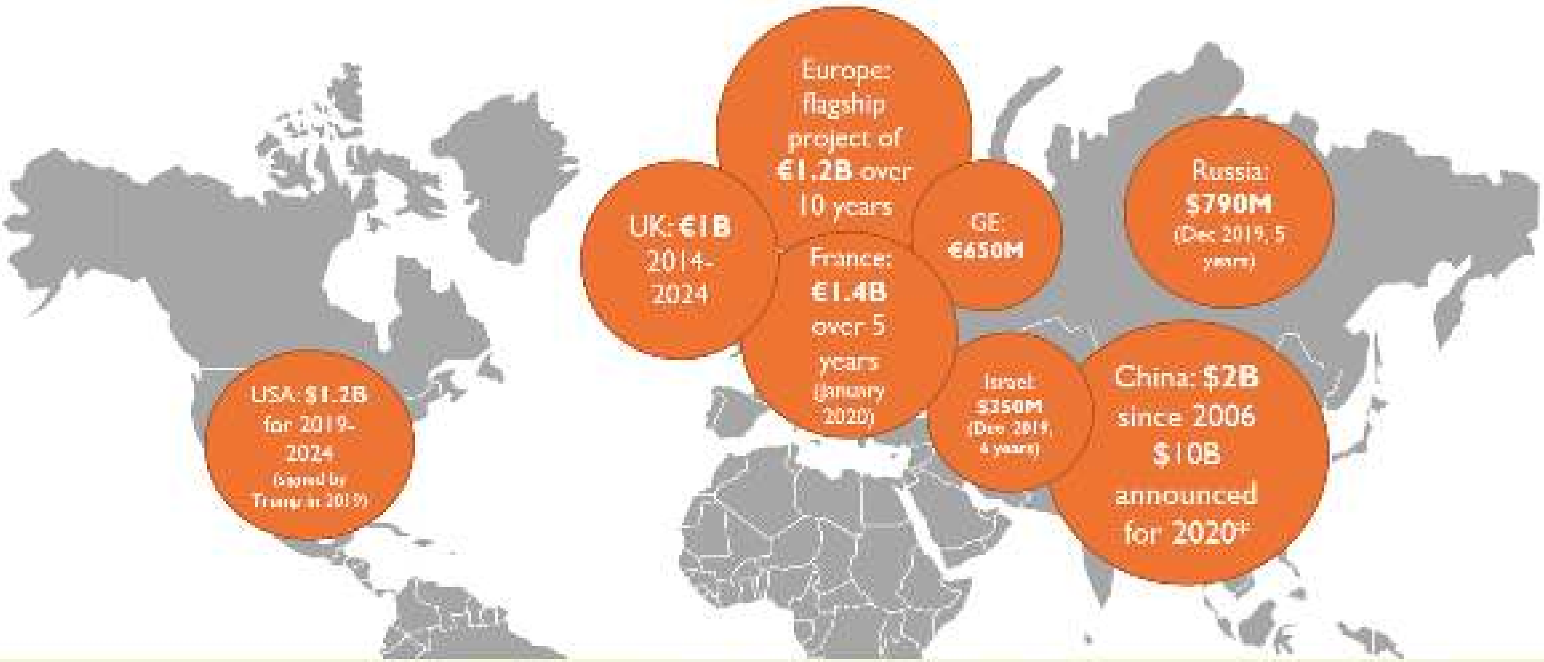
This two properties confer a sort of parallelism to a QC and bring the freedom to program designers, to do better than classical equivalents.





Quantum Technologies. Market and Technology Report 2020. Yole Developement

Quantum public investments

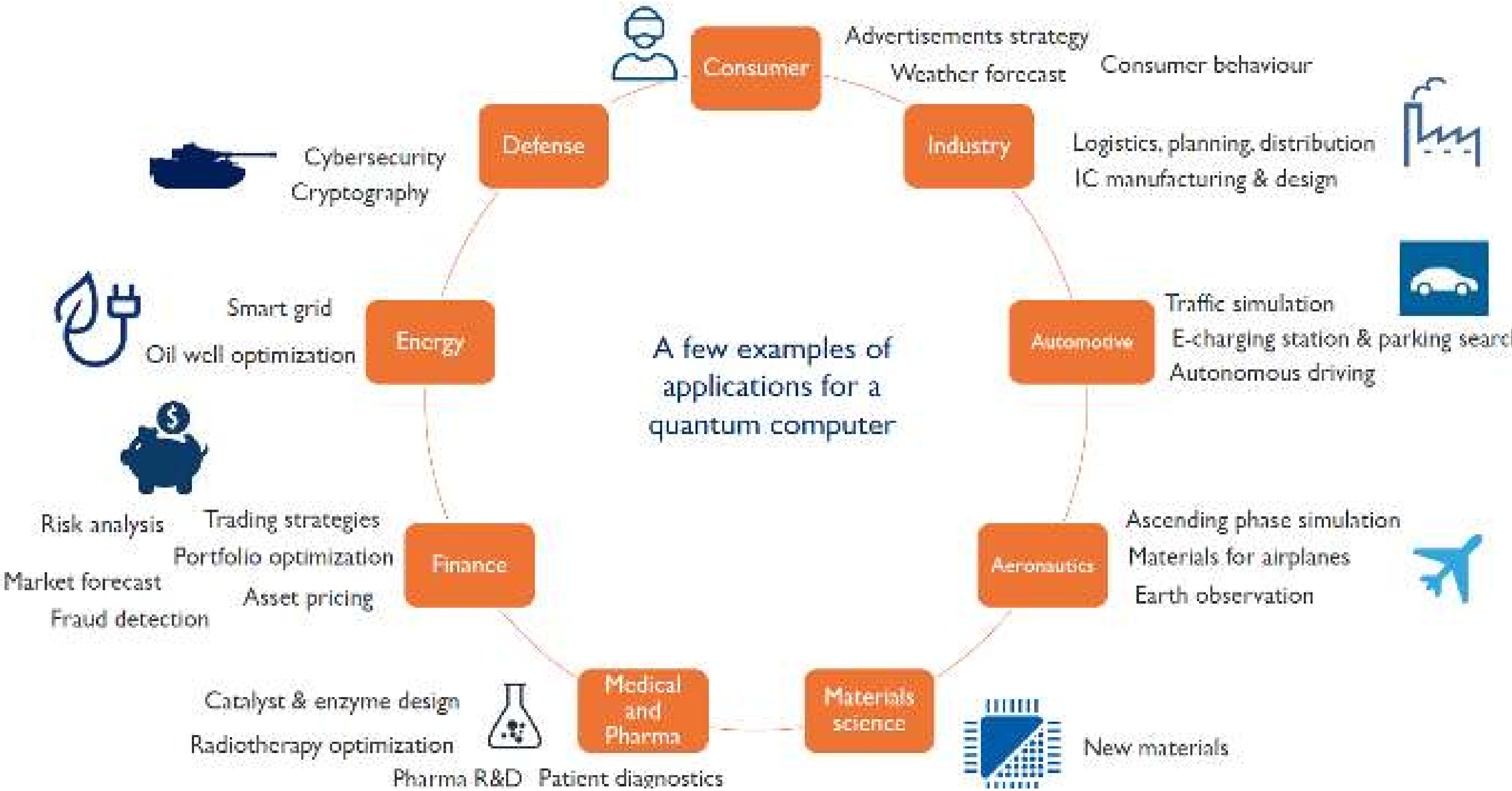


Europe is setting up a quantum effort to compete with US. IN Europe, UK was first to invest in QC (2013).
China is also involved in QC (Huawei, Alibaba ...). They are at 10-20 qubits development today, so late compared to US but China wants to be world quantum leader in 2024 ($10B investment). Map above shows major investments. There are also investment plans in Canada, Australia, Netherlands, Japan, Austria, Singapore.

More than $16B worldwide

Quantum Technologies. Market and Technology Report 2020. Yole Developement

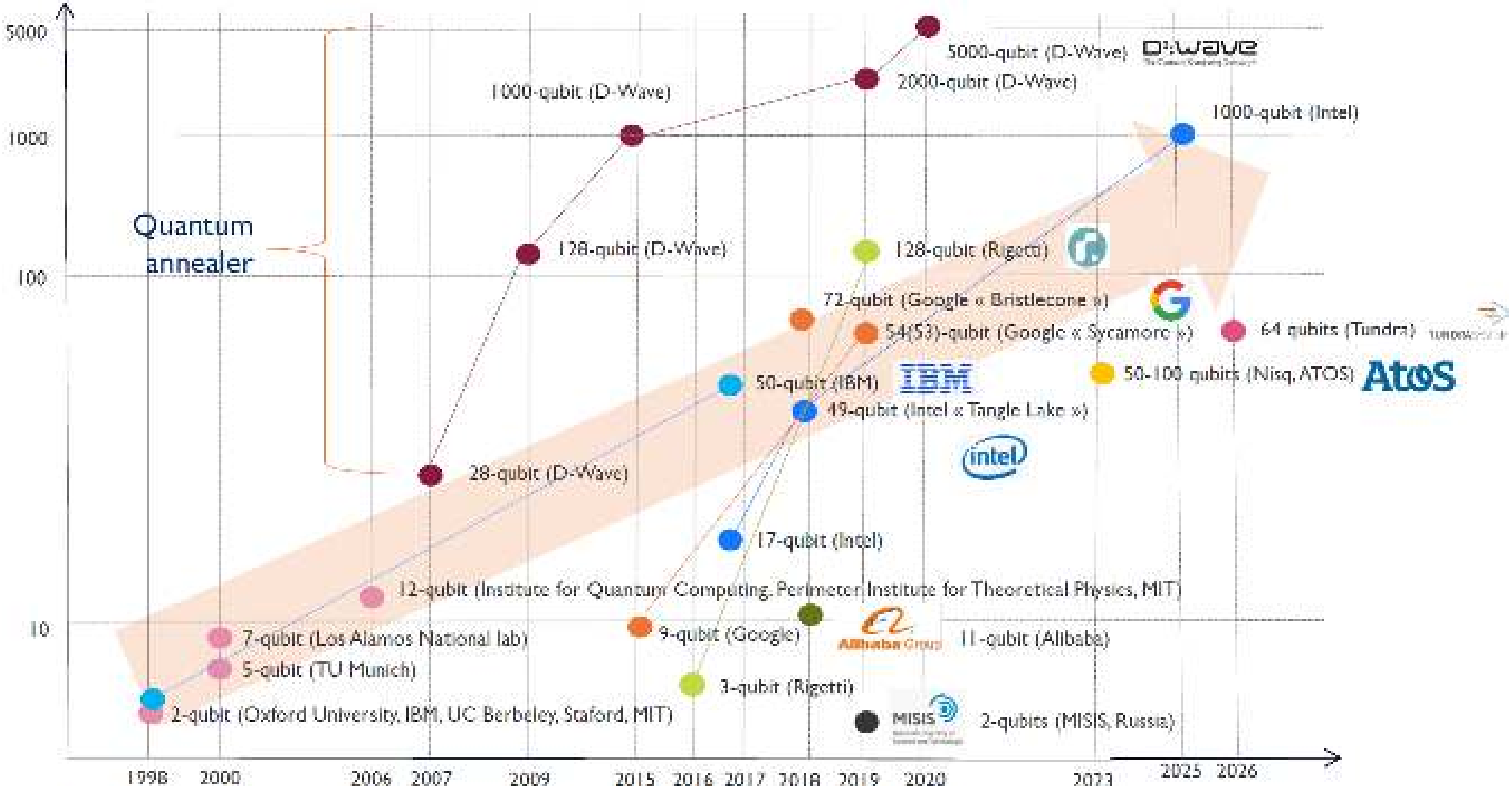
Quantum computing aplications



Quantum Technologies. Market and Technology Report 2020. Yole Developement

Physical qubits roadmap

Graph below shows physical qubits roadmap (to be remembered: for a quantum computer, 50 logic qubits minimum are required → it means 5000 physical qubits)



Quantum Technologies. Market and Technology Report 2020. Yole Developement

Game theory and economics

„Theory of Games and Economic Behaviour"
J. von Neumann and O. Morgenstern 1944

Nobel prizes for applications of GT to economics:

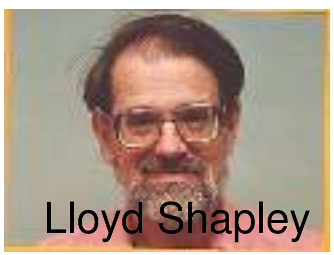
1994 Nash, Harsanyi, Selten "*for their pioneering analysis of equilibria in the theory of non-cooperative games*"

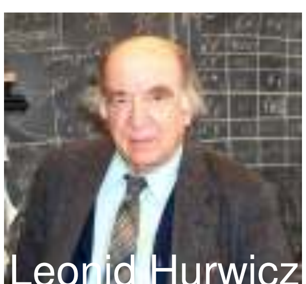
2005 Aumann i Schelling "*for having enhanced our understanding of conflict and cooperation through game-theory analysis"*

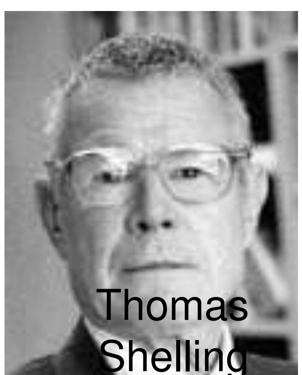
2007 Leonid Hurwicz, Eric Maskin i Roger Myerson „*for having laid the foundations of mechanism design theory*"

2012 Alvin Roth, Lloyd Shapley „*for the theory of stable allocations and the practice of market design*".

2020 Raul Milgrom, Robert Wilson "*for improvements to auction theory and inventions of new auction formats."*







2001 "Beatiful mind" movie about Nash

Games and probability distributions

We consider two player *games*

$$G = \left(N, \left\{S_X\right\}_{X \epsilon N}, \left\{P_X\right\}_{X \epsilon N}\right)$$

where:

$N = \{A, B\}$ is the set of players

$S_A = \{A_0, A_1\}$, $S_B = \{B_0, B_1\}$ are possible pure strategies

$P_X : S_A \times S_B \to \left\{v_{ij}^X \,\epsilon\, \mathbb{R} \,\middle|\, i,j = 0{,}1\right\}$, $X = A, B$, are payoff functions, represented by the game bimatrix

$$\begin{pmatrix} (v_{00}^A, v_{00}^B) & (v_{01}^A, v_{01}^B) \\ (v_{10}^A, v_{10}^B) & (v_{11}^A, v_{11}^B) \end{pmatrix}$$

Let

$$\Delta(S_A \times S_B) = \left\{\sum_{i,j=0,1} \sigma_{ij} A_i B_j \,\middle|\, \sigma_{ij} \geq 0, \sum_{i,j=0,1} \sigma_{ij} = 1\right\}$$

be the set of *probability distributions* over $S_A \times S_B$

Mixed strategies and Nash equilibria

If the set of probability distributions can be factorized

$$\begin{pmatrix} \sigma_{00} & \sigma_{01} \\ \sigma_{10} & \sigma_{11} \end{pmatrix} = \begin{pmatrix} \sigma_A \sigma_B & \sigma_A (1-\sigma_B) \\ (1-\sigma_A)\sigma_B & (1-\sigma_A)(1-\sigma_B) \end{pmatrix}$$

they define *mixed strategies* $\sigma_A, \sigma_B \epsilon\ [0,1]$.

The *mixed classical game* is

$$G^{mix} = (N, \Delta S_A, \Delta S_B, \Delta P_A, \Delta P_B)$$

where $\Delta(S_X) = \{\sigma_X X_0 + (1-\sigma_X) X_1 \mid 0 \leq \sigma_X \leq 1\} \equiv [0,1]$.

Mixed strategies form a subset of all probability distributions

$$\Delta S_A \times \Delta S_B \subset \Delta(S_A \times S_B)$$

The pair of strategies $(\sigma_A^*, \sigma_B^*)\ \epsilon\ \Delta S_A \times \Delta S_B$ is a Nash equilibrium

iff $\Delta P_A(\sigma_A^*, \sigma_B^*) \geq \Delta P_A(\sigma_A, \sigma_B^*)$ and $\Delta P_B(\sigma_A^*, \sigma_B^*) \geq \Delta P_B(\sigma_A^*, \sigma_B)$,

for each $\sigma_X\ \epsilon\ \Delta S_X$, $X = A, B$

Pareto optimality and correlated equilibria

A pair of strategies $(\sigma_A, \sigma_B) \epsilon S$ is *not Pareto optimal* in S if there exists another pair $(\sigma_A{}', \sigma_B{}') \epsilon S$ that is better for one of the players and not worse for the other. Otherwise $(\sigma_A, \sigma_B) \epsilon S$ is called *Pareto optimal*.

Probability distribution $\{\sigma_{ij}\}_{i,j=0,1}$ over set of strategies $(A_i, B_j)_{i,j=0,1}$ of the game G is a *correlated equilibrium* iff

$$\sum_{j=0,1} \sigma_{ij}\, v_{ij}^{A} \geq \sum_{j=0,1} \sigma_{ij}\, v_{-ij}^{A} \quad \text{and} \quad \sum_{j=0,1} \sigma_{ji}\, v_{ji}^{B} \geq \sum_{j=0,1} \sigma_{ji}\, v_{j(-i)}^{B}$$

where $-i \neq i$ is the index of the remaining strategy.

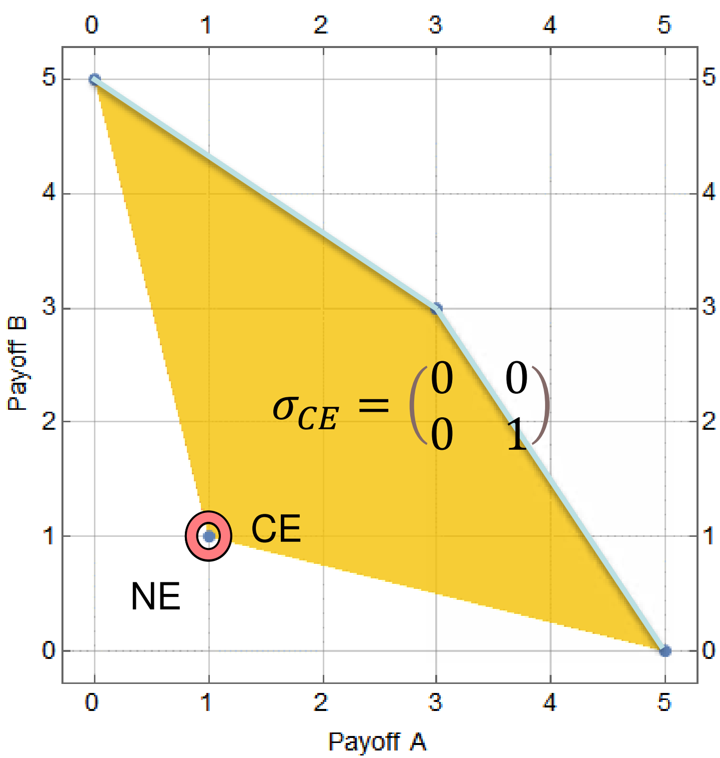
Efficiency of selected classical games

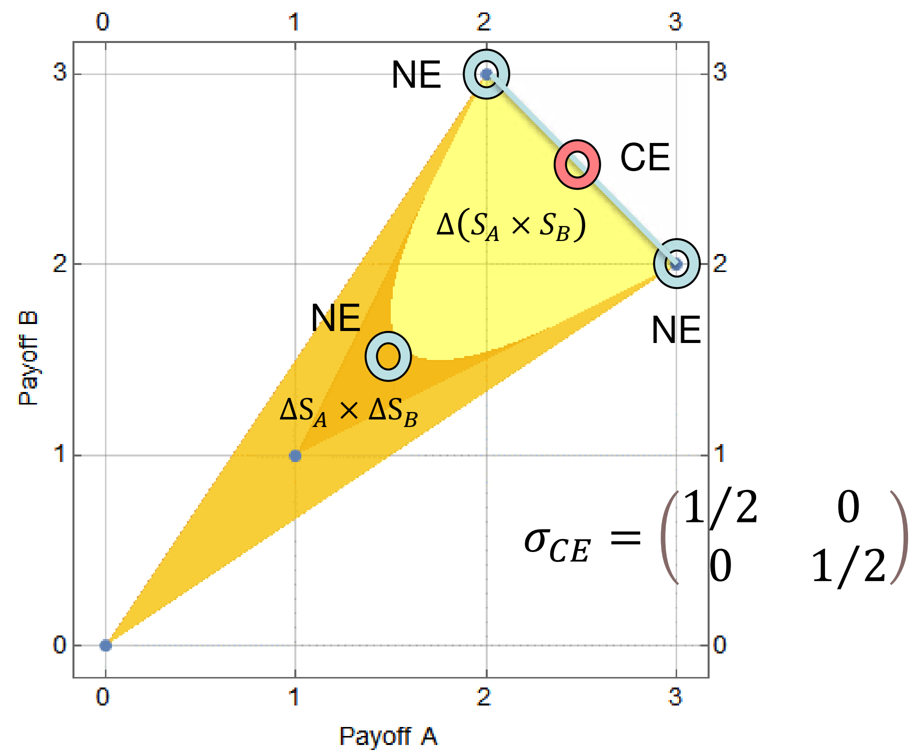
prisoner's dilemma		Bob	
		B_0	B_1
Alice	A_0	$(3,3)$	$(0,5)$
	A_1	$(5,0)$	$(1,1)$

battle of the sexes		Bob	
		B_0	B_1
Alice	A_0	$(3,2)$	$(1,1)$
	A_1	$(0,0)$	$(2,3)$

$$\sigma_{00} = \sigma_{01} = \sigma_{10} = 0$$
$$\sigma_{11} = 1$$

$$3\sigma_{00} \geq \sigma_{01}, \sigma_{00} \geq 3\sigma_{10}$$
$$3\sigma_{11} \geq \sigma_{01}, \sigma_{11} \geq 3\sigma_{10}$$





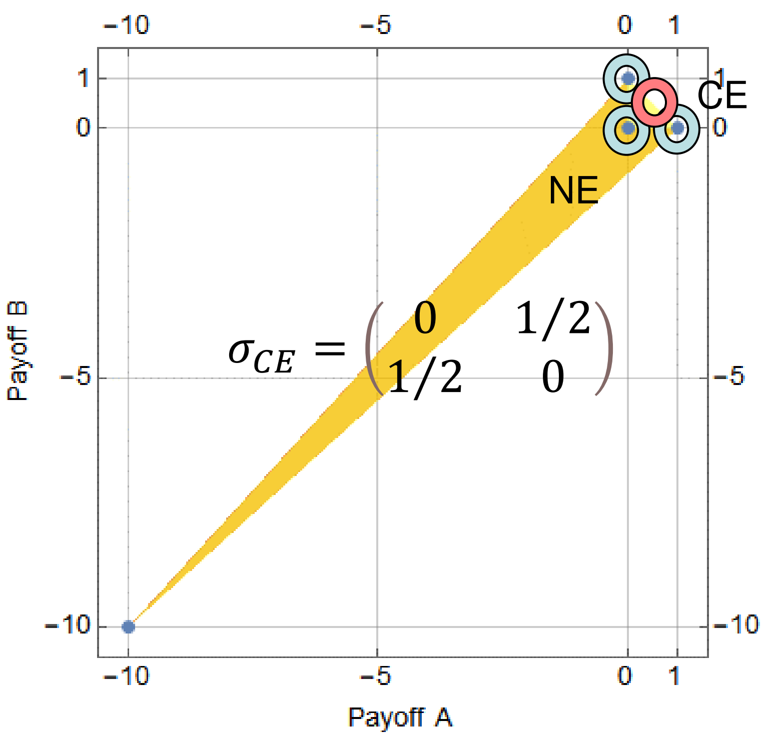
Efficiency of selected classical games

chicken		Driver B	
Driver A		B_0	B_1
	A_0	$(0,0)$	$(0,1)$
	A_1	$(1,0)$	$(-10,-10)$

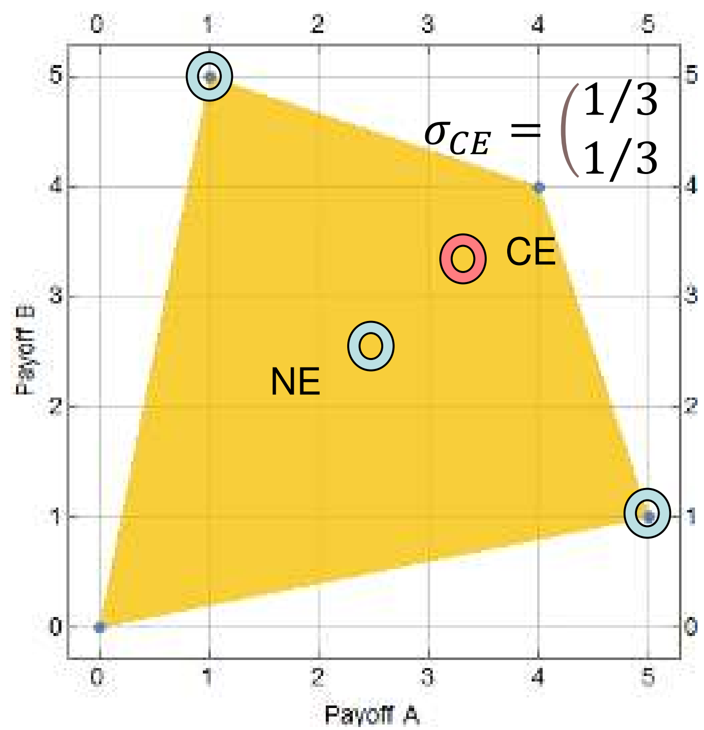
chicken 2		Player B	
Player A		B_0	B_1
	A_0	$(4,4)$	$(1,5)$
	A_1	$(5,1)$	$(0,0)$

$$\sigma_{00} \le 10\sigma_{01}, \sigma_{00} \le 10\sigma_{10}$$
$$10\sigma_{11} \le \sigma_{01}, 10\sigma_{11} \le \sigma_{10}$$

$$\sigma_{00} \le \sigma_{01}, \sigma_{00} \le \sigma_{10}$$
$$\sigma_{11} \le \sigma_{01}, \sigma_{11} \le \sigma_{10}$$



$$\sigma_{CE} = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix}$$



$$\sigma_{CE} = \begin{pmatrix} 1/3 & 1/3 \\ 1/3 & 0 \end{pmatrix}$$

Quantum game preliminaries

The standard quantum game in Eisert-Wilkens-Lewenstein quantization scheme is: (Eisert et al, PRL 83, 3077 (1999)

$$\Gamma_{EWL} = (N, \{U_X\}_{X \epsilon N}, \{\Pi_X\}_{X \epsilon N})$$

where:

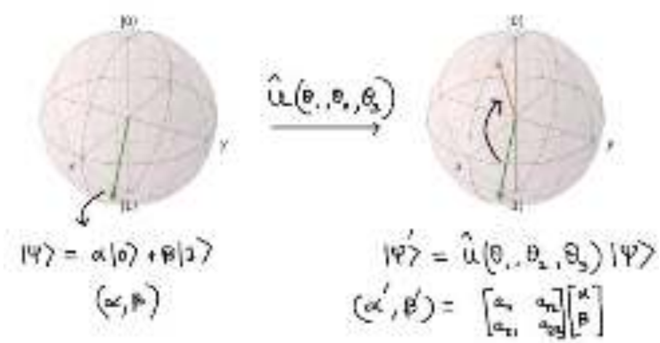
$N = \{A, B\}$ is the set of players

The unitary transformations $\hat{U}_A = \hat{U}(\theta_A, \alpha_A, \beta_A)$, $\hat{U}_B = \hat{U}(\theta_B, \alpha_B, \beta_B)$

$$\hat{U}(\theta_X, \alpha_X, \beta_X) = \begin{pmatrix} e^{i\alpha_X} cos\frac{\theta_X}{2} & ie^{i\beta_X} sin\frac{\theta_X}{2} \\ ie^{-i\beta_X} sin\frac{\theta_X}{2} & e^{-i\alpha_X} cos\frac{\theta_X}{2} \end{pmatrix},$$

$\theta_X \in [0, \pi], \alpha_X, \beta_X \in [0, 2\pi], X = A, B$

are quantum strategies.



Quantum game payoffs

$\Pi_X: SU(2) \times SU(2) \to \mathbb{R}$ are payoff functions defined by:

$$\Pi_X\left(\hat{U}_A, \hat{U}_B, \gamma\right) = \sum_{k,l=0}^{1} v_{k,l}^X \left|\langle \Psi_{k,l}(\gamma) \middle| U_A \otimes U_B \middle| \Psi(\gamma) \rangle\right|^2, \; X = A, B$$

$$|\Psi_{k,l}(\gamma)\rangle = C_k \otimes C_l |\Psi(\gamma)\rangle$$

In case of a fully quantum case $\gamma = \pi/2$:

$$\Pi_X(\hat{U}_A, \hat{U}_B) = \sum_{k,l=0,1} |p_{kl}|^2 \, v_{kl}^X, \quad X = A, B,$$

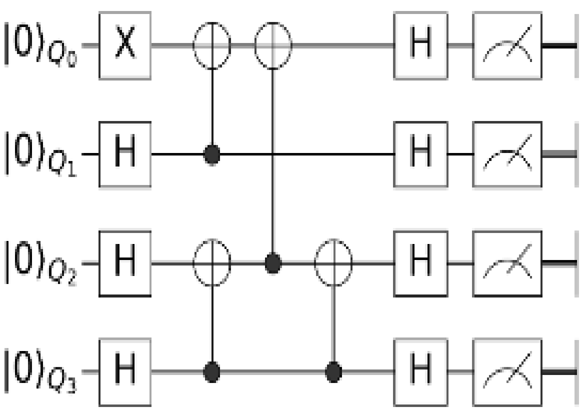
where:

$$|p_{00}|^2 = \cos\frac{\theta_A}{2}\cos\frac{\theta_B}{2}\cos(\alpha_A + \alpha_B) + \sin\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\sin(\beta_A + \beta_B),$$

$$|p_{01}|^2 = \cos\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\cos(\alpha_A - \beta_B) + \sin\frac{\theta_A}{2}\cos\frac{\theta_B}{2}\sin(\alpha_B - \beta_A),$$

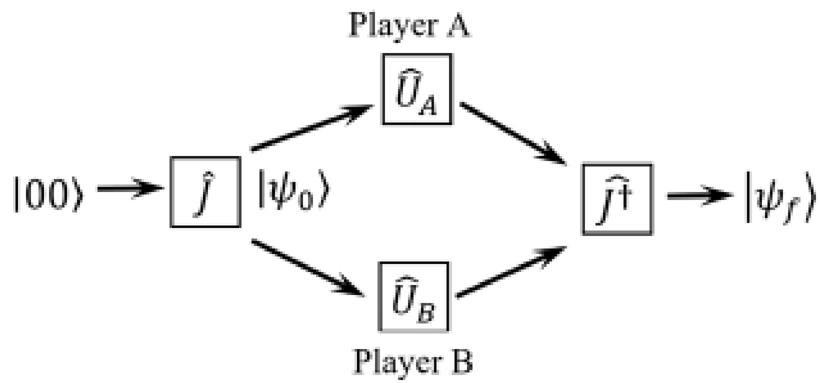
$$|p_{10}|^2 = \cos\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\sin(\alpha_A - \beta_B) + \sin\frac{\theta_A}{2}\cos\frac{\theta_B}{2}\cos(\alpha_B - \beta_A),$$

$$|p_{11}|^2 = \cos\frac{\theta_A}{2}\cos\frac{\theta_B}{2}\sin(\alpha_A + \alpha_B) - \sin\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\cos(\beta_A + \beta_B).$$



EWL approach

The quantum EWL approach to the game is



where: $|00\rangle$ is the initial state

$\hat{J} = \frac{1}{\sqrt{2}}(\hat{I} + i\sigma_x \otimes \sigma_x), J^\dagger$ are the entangling, disentangling operators,

$$\hat{U}_X(\theta_X, \alpha_X, \beta_X) = \begin{pmatrix} e^{i\alpha_X} \cos\frac{\theta_X}{2} & ie^{i\beta_X} \sin\frac{\theta_X}{2} \\ ie^{-i\beta_X} \sin\frac{\theta_X}{2} & e^{-i\alpha_X} \cos\frac{\theta_X}{2} \end{pmatrix}, X = A, B,$$

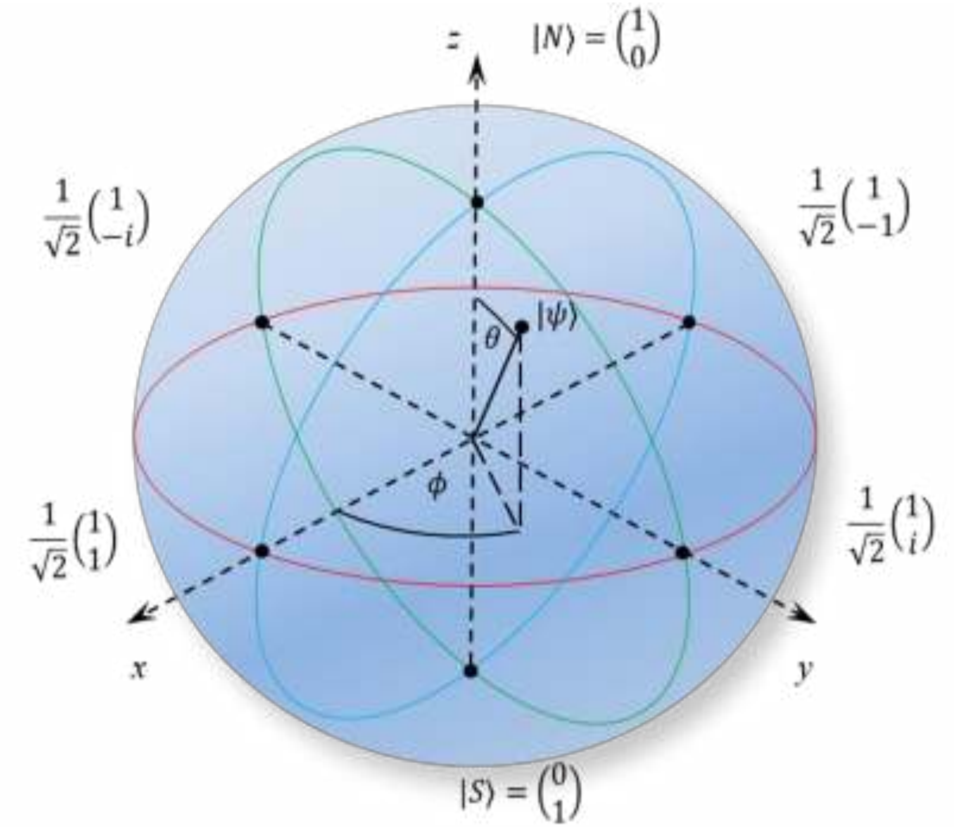
$\left|\psi_f\right\rangle = \sum_{i,j=0,1} p_{ij} \; |ij\rangle$, is the final state defining the game payoffs

Quantum Pauli strategies

Strategies $\widehat{U}_A = \widehat{U}(\theta_A, \alpha_A, \beta_A)$ and $\widehat{U}_B = \widehat{U}(\theta_B, \alpha_B, \beta_B)$,

$$\widehat{U}_X(\theta_X, \alpha_X, \beta_X) = \begin{pmatrix} e^{i\alpha_X}\cos\frac{\theta_X}{2} & ie^{i\beta_X}\sin\frac{\theta_X}{2} \\ ie^{-i\beta_X}\sin\frac{\theta_X}{2} & e^{-i\alpha_X}\cos\frac{\theta_X}{2} \end{pmatrix},$$

are generated by Pauli strategies:



$$\widehat{P_0} = \widehat{U}(0,0,\beta) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\widehat{P_x} = \widehat{U}(\pi, \alpha, \pi) = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix},$$

$$\widehat{P_y} = \widehat{U}(\pi, \alpha, \pi/2) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},$$

$$\widehat{P_z} = \widehat{U}(0, \pi/2, \beta) = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}.$$

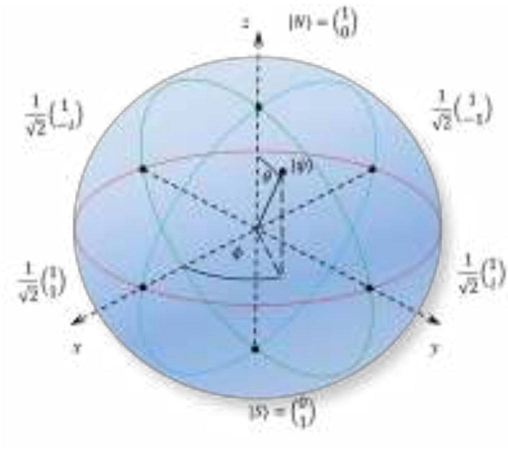
where

$$\widehat{\sigma_x} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \widehat{\sigma_y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \widehat{\sigma_z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

are Pauli matrices

Classical limit of the quantum game

Le us assume $\alpha = \beta = 0$, in this case



$$\hat{U}(\theta, 0, 0) = \cos\frac{\theta}{2}\hat{I} + i\sin\frac{\theta}{2}\sigma_x$$

is equivalent to the classical mixed strategy

		B	
A		$cos^2\frac{\theta_B}{2}$	$sin^2\frac{\theta_B}{2}$
	$cos^2\frac{\theta_A}{2}$	(a_{00}, b_{00})	(a_{01}, b_{01})
	$sin^2\frac{\theta_A}{2}$	(a_{10}, b_{10})	(a_{11}, b_{11})

$\theta_B = 0$ $\theta_B = \pi$

and the payoffa are

$$\$_{A(\boldsymbol{B})} = a(b)_{00}\cos^2\frac{\theta_A}{2}\cos^2\frac{\theta_B}{2} + a(b)_{01}\cos^2\frac{\theta_A}{2}\sin^2\frac{\theta_B}{2}$$
$$+a(b)_{10}\sin^2\frac{\theta_A}{2}\cos^2\frac{\theta_B}{2} + a(b)_{11}\sin^2\frac{\theta_A}{2}\sin^2\frac{\theta_B}{2}$$

EWL with Frąckiewicz-Pykacz parameterization

Let us restrict the set of quantum strategies to

$$\hat{U}_X(\theta_X, \phi_X) = \begin{pmatrix} e^{-i\phi_X}\cos\frac{\theta_X}{2} & -e^{-i\phi_X}\sin\frac{\theta_X}{2} \\ e^{i\phi_X}\sin\frac{\theta_X}{2} & e^{i\phi_X}\cos\frac{\theta_X}{2} \end{pmatrix}$$

$$\widehat{P_0} = \hat{U}(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$
$$\widehat{P_x} = \hat{U}\left(\pi, \frac{3\pi}{2}\right) = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix},$$
$$\widehat{P_y} = \hat{U}(\pi, 0) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},$$
$$\widehat{P_z} = \hat{U}\left(0, \frac{3\pi}{2}\right) = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}.$$

- In this parameterization, there are additional Nash equilibria in pure strategies
- F-P parametrization is invariant with respect to strongly isomorphic transformation of input games

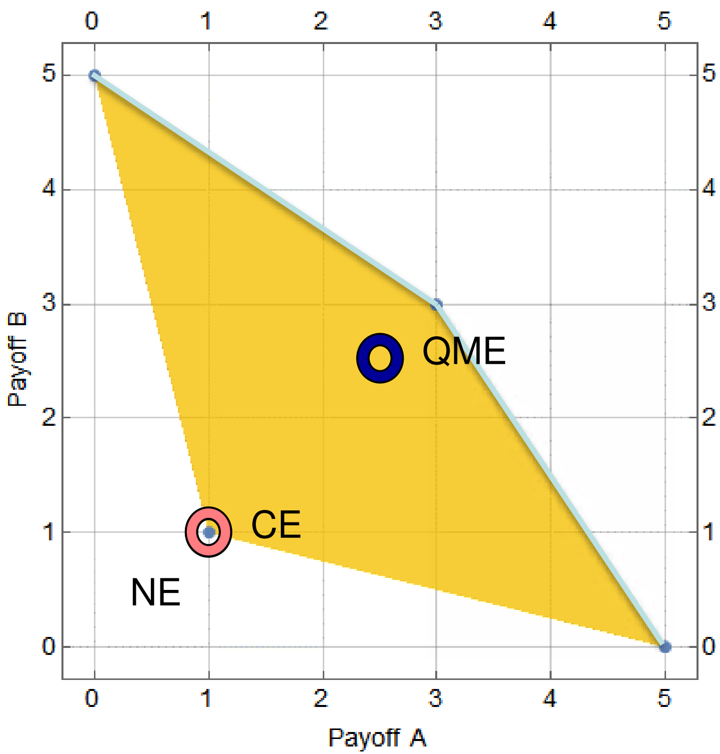
Quantum Mixed Equilibria

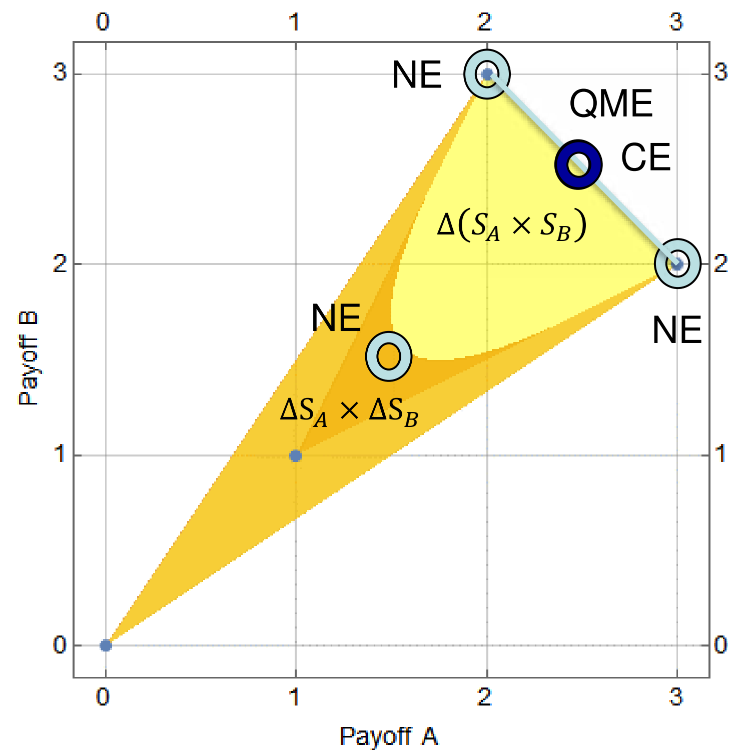
prisoner's dilemma		Bob	
		B_0	B_1
Alice	A_0	$(3,3)$	$(0,5)$
	A_1	$(5,0)$	$(1,1)$

battle of the sexes		Bob	
		B_0	B_1
Alice	A_0	$(3,2)$	$(1,1)$
	A_1	$(0,0)$	$(2,3)$

$$\sigma^A = \left(\frac{1}{2}, 0, 0, \frac{1}{2}\right), \sigma^B = \left(0, \frac{1}{2}, \frac{1}{2}, 0\right)$$

$$\sigma^A = \sigma^B = \left(\frac{1}{2}, 0, 0, \frac{1}{2}\right)$$





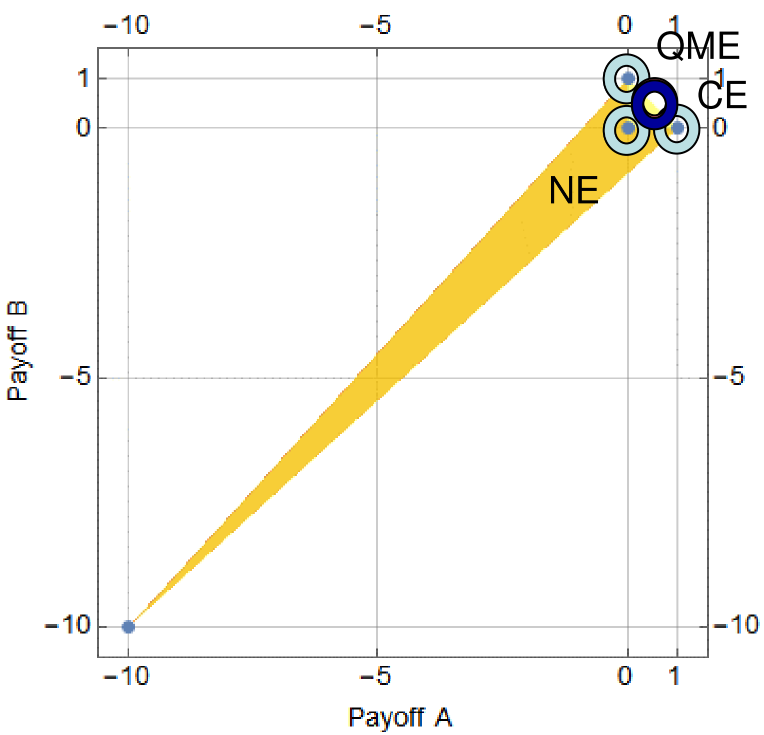
Quantum Mixed Equilibria

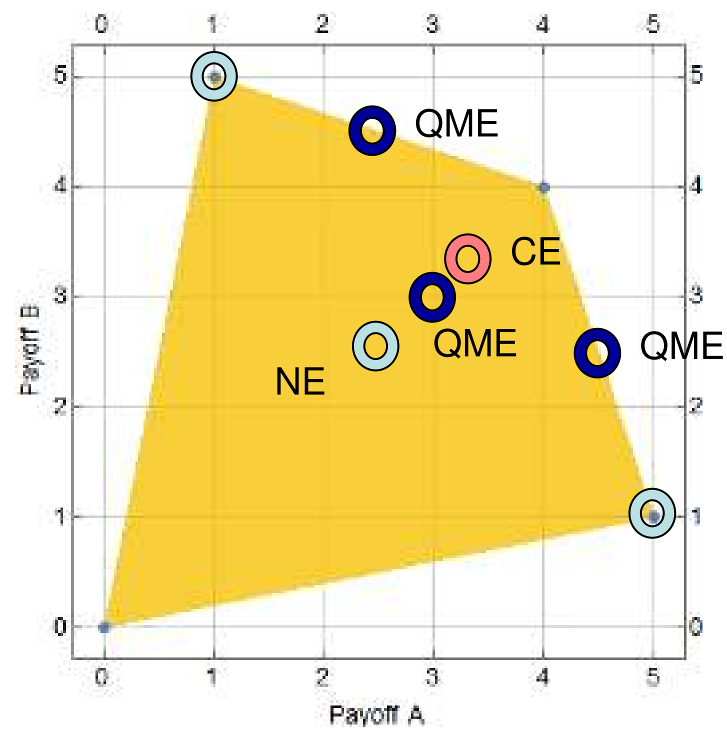
chicken		Driver B	
Driver A		B_0	B_1
	A_0	$(0,0)$	$(0,1)$
	A_1	$(1,0)$	$(-10,-10)$

chicken 2		Player B	
Player A		B_0	B_1
	A_0	$(4,4)$	$(1,5)$
	A_1	$(5,1)$	$(0,0)$

$$\sigma^A = \left(\frac{1}{2}, 0, 0, \frac{1}{2}\right), \sigma^B = \left(0, \frac{1}{2}, \frac{1}{2}, 0\right)$$

$$\sigma^A = \left(\frac{1}{2}, 0, \frac{1}{2}, 0\right), \sigma^B = \left(\frac{1}{2}, \frac{1}{2}, 0, 0\right)$$

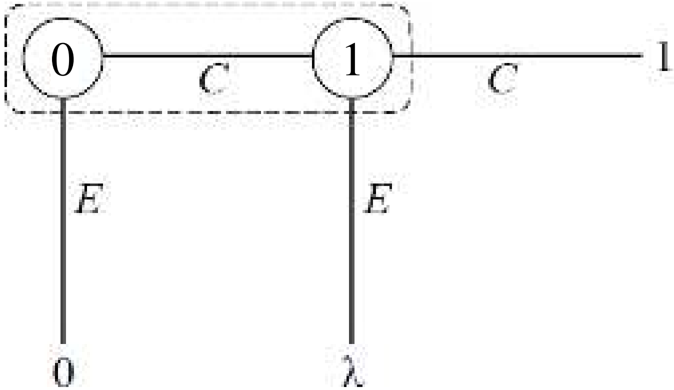




The absentminded driver paradox *)

Decision problem with imperfect recall

- A decision maker is planning a trip home
- The highway have two consecutive exits 1 and 2.
- Driver can Continue or Exit
- Payoffs at exits are:
  - 0 – catastrophic area
  - λ – home, $(\lambda > 2)$
  - 1 – motel



When he arrives at an intersection, the driver cannot tell whether the intersection leads to the first or the second exit. The expected payoff is

$$E(p) = p(1-p)\lambda + p^2$$

where p is the probability of continuing at the intersection

*) Piccione M., Rubinstein A. On the interpretation of decision problems with imperfect recall, Game Econ. Behav. 20, 3-24 (1997).

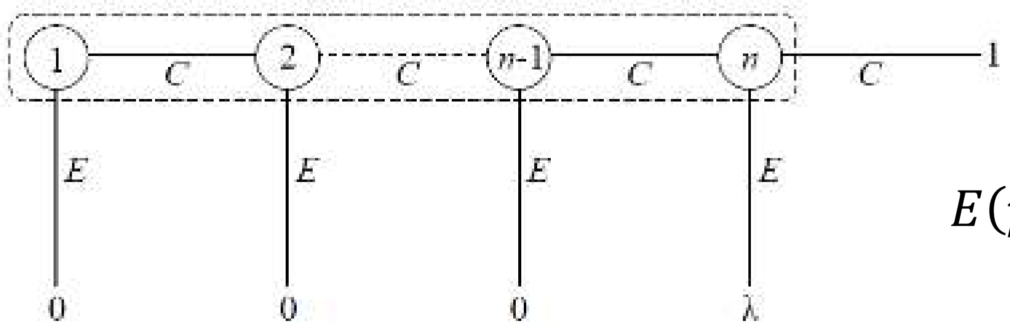
The absentminded driver with n intersections

The optimal strategy is:

$$p_{max} = \begin{cases} 1 & \text{if } \lambda \le 2 \\ \dfrac{\lambda}{2(\lambda - 1)} & \text{if } \lambda > 2 \end{cases}$$

and the payoff for $\lambda > 2$ is $E_{max} = \frac{\lambda^2}{4(\lambda-1)}$, i.e. for $\lambda = 4$ $E_{max} = \frac{4}{3}$

The absent-minded driver problem with n intersections:



$$E(p) = p^{n-1}(1 - p)\lambda + p^n$$

$$p_{max} = \begin{cases} 1 & \text{if } \lambda \le n \\ \dfrac{(n-1)\lambda}{n(\lambda - 1)} & \text{if } \lambda > n \end{cases}$$

$$E_{max}(\lambda = 2n) = 2^n \left(\frac{n-1}{2n-1}\right)^{n-1} \xrightarrow[n\to\infty]{} \frac{2}{\sqrt{e}}$$

Quantum absentminded driver

Let

$$|\Psi(\gamma)\rangle = \cos\left(\frac{\gamma}{2}\right)|00\rangle + i\sin\left(\frac{\gamma}{2}\right)|11\rangle$$

be the arbitrary initial state with the (entanglement) parameter

$$\gamma \in [0, \pi/2]$$

The payoff bimatrix of the classical game is

$$\begin{pmatrix} (0,0) & (0,0) \\ (\lambda,\lambda) & (1,1) \end{pmatrix}$$

and

$$C_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad C_1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

represent the two possible actions of the driver „exit" and „continue"

$$(|\Psi_{k,l}(\gamma)\rangle = C_k \otimes C_l |\Psi(\gamma)\rangle)$$

P. Frąckiewicz, K. Rycerz, M. Szopa, *Quantum absentminded driver problem revisited*, subm. to Quantum Inf. Processing

Quantum absentminded driver

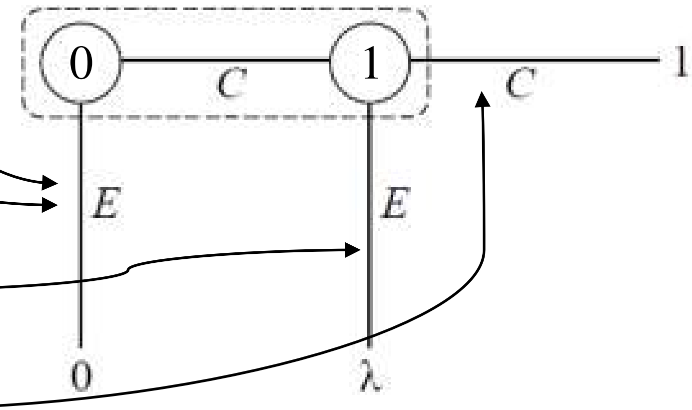
The basis vectors are:

$$|\Psi_{0,0}(\gamma)\rangle = \cos\left(\frac{\gamma}{2}\right)|00\rangle - i\sin\left(\frac{\gamma}{2}\right)|11\rangle$$

$$|\Psi_{0,1}(\gamma)\rangle = i\cos\left(\frac{\gamma}{2}\right)|01\rangle - \sin\left(\frac{\gamma}{2}\right)|10\rangle$$

$$|\Psi_{1,0}(\gamma)\rangle = -\sin\left(\frac{\gamma}{2}\right)|01\rangle + i\cos\left(\frac{\gamma}{2}\right)|10\rangle$$

$$|\Psi_{1,1}(\gamma)\rangle = i\sin\left(\frac{\gamma}{2}\right)|00\rangle - \cos\left(\frac{\gamma}{2}\right)|11\rangle$$



Using this, we get payoffs functions of the absentminded driver:

$$\left|\langle\Psi_{00}(\gamma)|U^{\otimes 2}|\Psi(\gamma)\rangle\right| = \left|e^{2i\alpha}\sin 2\beta\sin\gamma\sin^2\frac{\theta}{2} + \frac{1}{2}\cos^2\frac{\theta}{2}\left(e^{4i\alpha}(\cos\gamma+1) - \cos\gamma + 1\right)\right|,$$

$$\left|\langle\Psi_{01}(\gamma)|U^{\otimes 2}|\Psi(\gamma)\rangle\right| = \frac{1}{2}\left|\left(e^{2i\alpha}\cos\frac{\gamma}{2} + ie^{2i\beta}\sin\frac{\gamma}{2}\right)\sin\theta\right|,$$

$$\left|\langle\Psi_{10}(\gamma)|U^{\otimes 2}|\Psi(\gamma)\rangle\right| = \frac{1}{2}\left|\left(e^{2i\alpha}\cos\frac{\gamma}{2} + ie^{2i\beta}\sin\frac{\gamma}{2}\right)\sin\theta\right|,$$

$$\left|\langle\Psi_{11}(\gamma)|U^{\otimes 2}|\Psi(\gamma)\rangle\right| = \left|e^{2i\alpha}\sin^2\frac{\theta}{2}\left(\cos^2\frac{\gamma}{2} + e^{4i\beta}\sin^2\frac{\gamma}{2}\right) + \frac{i}{2}\left(e^{4i\alpha}-1\right)e^{2i\beta}\sin\gamma\cos^2\frac{\theta}{2}\right|,$$

P. Frąckiewicz, K. Rycerz, M. Szopa, *Quantum absentminded driver problem revisited*, subm. to Quantum Inf. Processing

Quantum absentminded driver

The payoff of the absentminded driver is therefore:

$$v_{\Psi}(U,\gamma) = \frac{1}{4}\lambda\left|\left(\mathrm{e}^{2\mathrm{i}\alpha}\cos\frac{\gamma}{2} + \mathrm{i}\mathrm{e}^{2\mathrm{i}\beta}\sin\frac{\gamma}{2}\right)\sin\theta\right|^2 + \left|\frac{1}{2}\mathrm{i}\mathrm{e}^{2\mathrm{i}\beta}\left(-1+\mathrm{e}^{4\mathrm{i}\alpha}\right)\cos^2\frac{\theta}{2}\sin\gamma + \mathrm{e}^{2\mathrm{i}\alpha}\left(\cos^2\frac{\gamma}{2} + \mathrm{e}^{4\mathrm{i}\beta}\sin^2\frac{\gamma}{2}\right)\sin^2\frac{\theta}{2}\right|^2$$

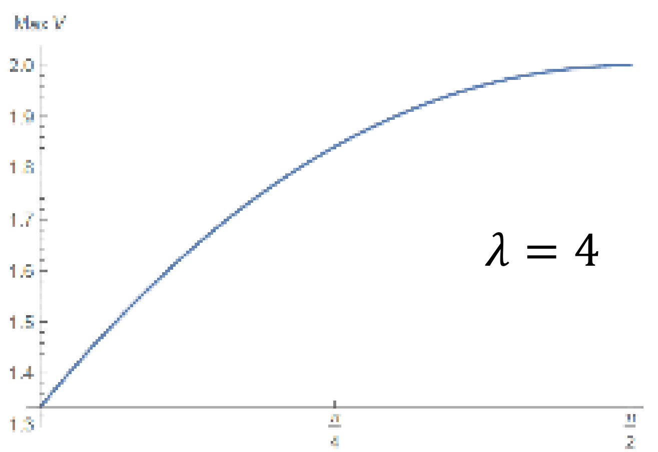
And its maximum is:

$$\max_{U\in \mathsf{SU}(2),\gamma\in[0,\frac{\pi}{2}]} v_{\Psi}(U,\gamma) = \frac{\lambda}{2} \quad \text{for} \quad \gamma = \theta = \frac{\pi}{2} \text{ and } \beta - \alpha = (n + \tfrac{3}{4})\pi$$

The dependence of $\max v_{\Psi}(U,\gamma)$ on the entanglement γ is:

Quantum value: $E_{max} = 2$

Classic value: $E_{max} = \frac{4}{3}$



P. Frąckiewicz, K. Rycerz, M. Szopa, *Quantum absentminded driver problem revisited*, subm. to Quantum Inf. Processing

Different entangled initial state

Let us assume different initial state:

$$|\Phi(\gamma)\rangle = \cos\left(\frac{\gamma}{2}\right)|01\rangle + i\sin\left(\frac{\gamma}{2}\right)|10\rangle$$

The payoff is:

$$\begin{aligned} v_\Phi(U,\gamma) &= \lambda\left|\langle\Phi_{10}(\gamma)|U^{\otimes 2}|\Psi(\gamma)\rangle\right|^2 + \left|\langle\Phi_{11}(\gamma)|U^{\otimes 2}|\Psi(\gamma)\rangle\right|^2 \\ &= \frac{\lambda}{2}\left|\left(\cos\frac{\gamma}{2} - ie^{2i(\alpha+\beta)}\sin\frac{\gamma}{2}\right)\sin\theta\right|^2 + \sin^2\frac{\theta}{2}. \end{aligned}$$

With the maximum:

$$\max_{U\in SU(2),\gamma\in[0,\frac{\pi}{2}]} v_\Phi(U,\gamma) = \max_{\theta\in[0,\pi]} v_\Phi\left(U\left(\theta,\alpha,\frac{\pi}{4}-\alpha\right),\frac{\pi}{2}\right) = \max_{\theta\in[0,\pi]}\left(\sin^2\theta + \sin^4\frac{\theta}{2}\right)$$

at $\theta = \arccos\frac{1}{1-2\lambda}$

and is equal to:

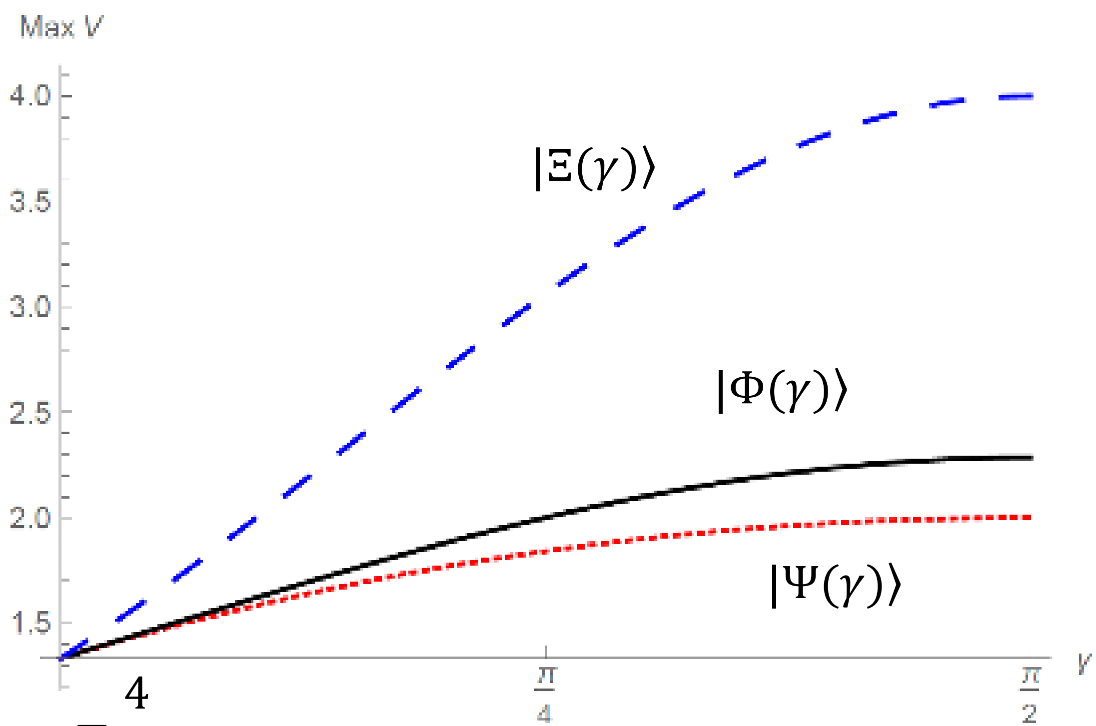
$$v_\Phi\left(U\left(\arccos\frac{1}{1-2\lambda},\alpha,\frac{\pi}{4}-\alpha\right),\frac{\pi}{2}\right) = \frac{\lambda^2}{2\lambda-1} \qquad E_{max} = 2\frac{2}{3}$$

Unentangled initial state

Let us assume unentangled initial state:

$$|\Xi(\gamma)\rangle = \cos\left(\frac{\gamma}{2}\right)|00\rangle + i\sin\left(\frac{\gamma}{2}\right)|10\rangle$$

the maximal payoffs for $\lambda = 4$ corresponding to different initial states are:



Classic value: $E_{max} = \frac{4}{3}$

P. Frąckiewicz, K. Rycerz, M. Szopa, *Quantum absentminded driver problem revisited*, subm. to Quantum Inf. Processing

Absentminded driver with n-intersections

The initial state is:

$$|\Psi\rangle = \left(\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right)^{\otimes n-1} \otimes |0\rangle$$

And the orthogonal basis for n-intersections is

$$\{V_1 \otimes V_2 \otimes \cdots \otimes V_n|\Psi\rangle, V_i \in \{1, i\sigma_x\}\}$$

The optimal strategy for this problem is

$$|\Psi_f\rangle = U(0, \pi/2, 0)^{\otimes n}|\Psi\rangle$$

because

$$\langle\Psi_{11\ldots 10}|\Psi_f\rangle = (-i)^{n-1}i$$

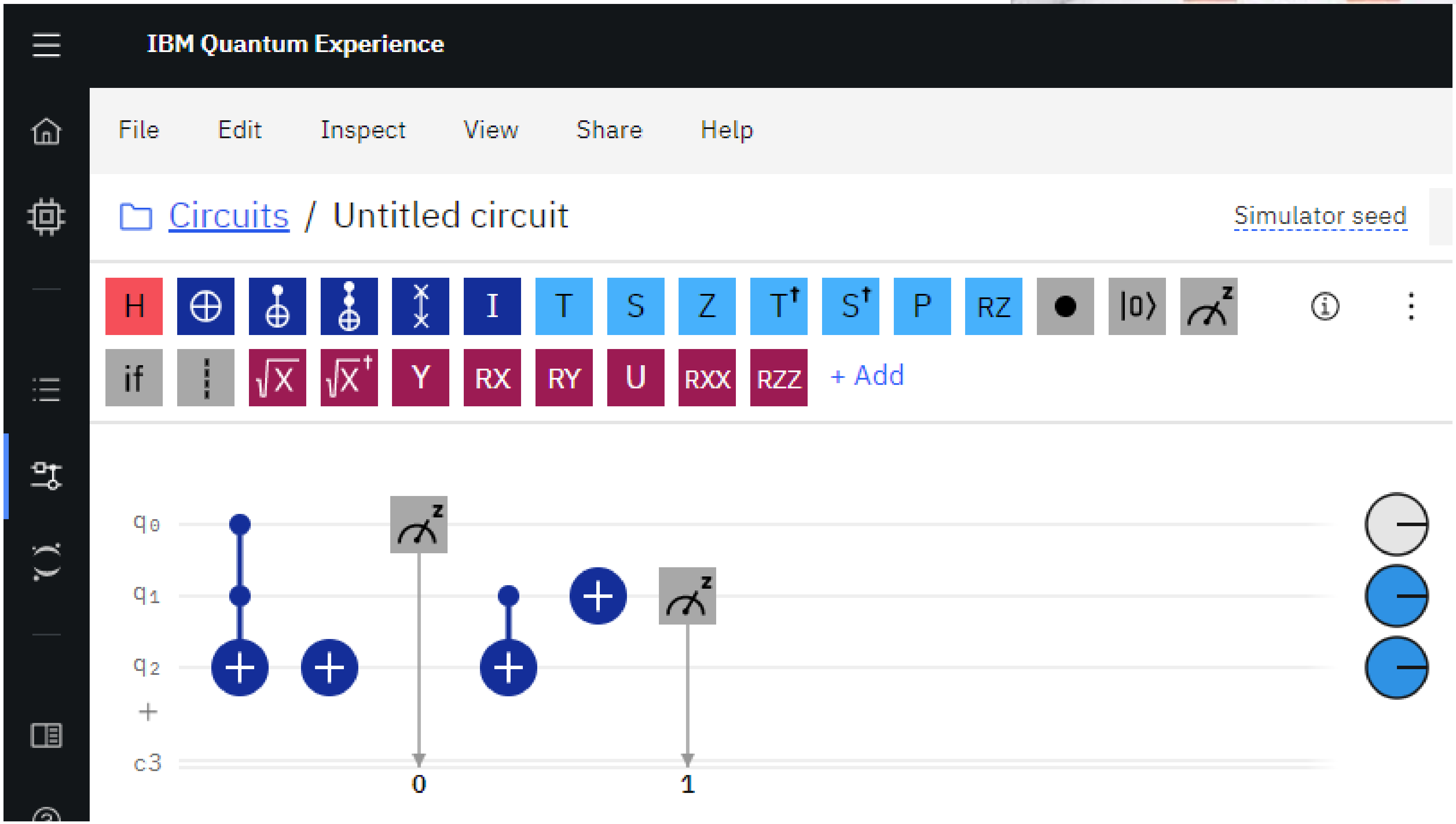
and, as a result:

$$u(U(0, \pi/2, 0)^{\otimes n}) = \lambda$$

P. Frąckiewicz, K. Rycerz, M. Szopa, *Quantum absentminded driver problem revisited*, subm. to Quantum Inf. Processing

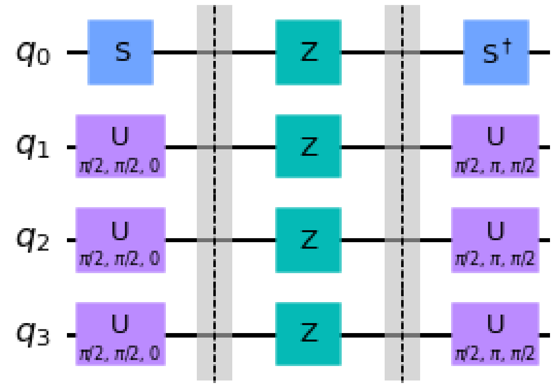
Quantum Computer

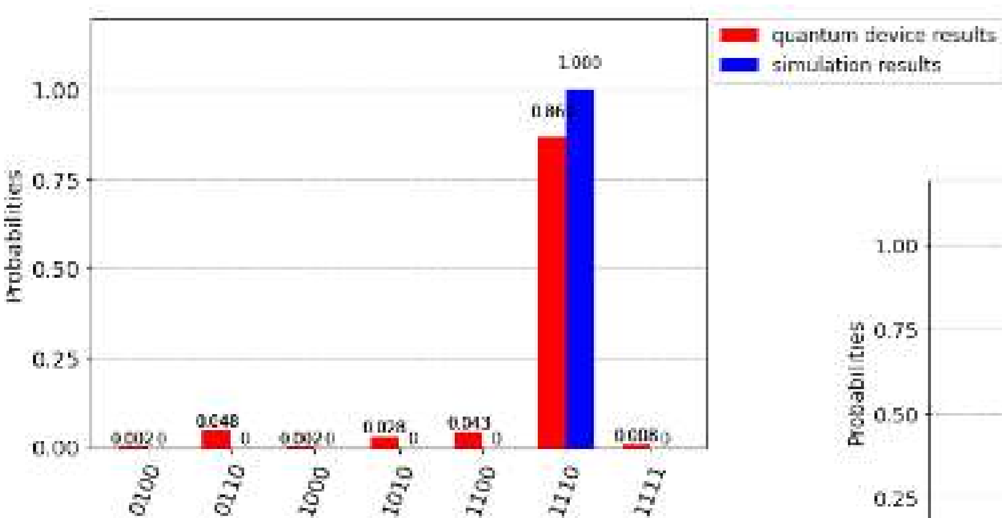
https://quantum-computing.ibm.com/

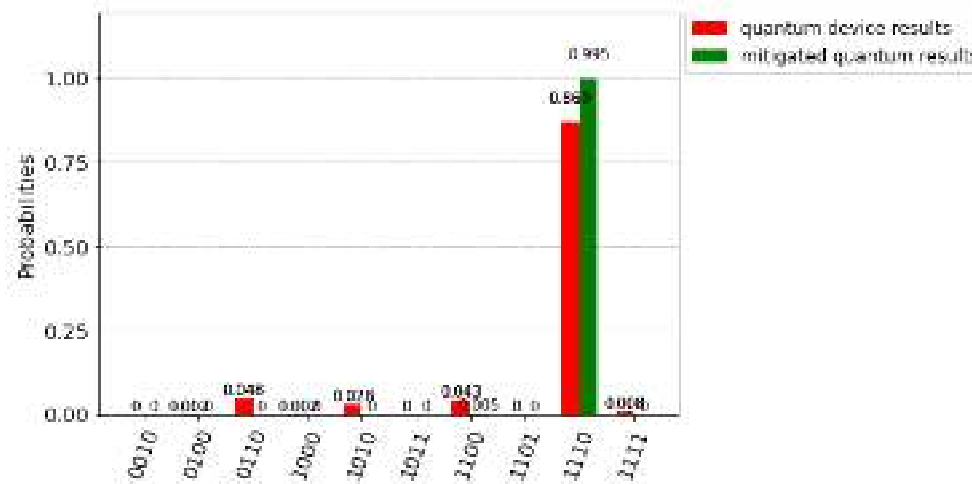


Quantum absentminded driver on IBM-Q

P. Frąckiewicz, K. Rycerz, M. Szopa;
Quantum absentminded driver problem revisited,
subm. to Quantum Inf. Processing







P. Frąckiewicz, K. Rycerz, M. Szopa, *Quantum absentminded driver problem revisited*, subm. to Quantum Inf. Processing

Conclusions

1. Correlated equilibria significantly improve paretoefficiency of Nash equilibria and they can be obtained in quantum games
2. Quantum games give players new strategies not available in classic games and strongly depend on the parameterization used
3. Nash equilibria of quantum in mixed strategies are close to paretoefficiency of correlated equilibria
4. FP parameterization provides a strong isomorphism of the quantum game and gives the same Nash equilibria in mixed strategies as full SU(2) parameterization of EWL
5. The entanglement of initial state is not necessary to define the quantum absentminded driver model, the key issue is the coherence of quantum evolution
6. The expected payoff of QAD is an increasing function of the entanglement γ and reaches the highest possible value of λ for separable initial state