15.05.2023 KASZTANOWE WIRTUALNE SEMINARIUM Z BADAŃ OPERACYJNYCH

# POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS – CASE OF PURE AND MIXED STRATEGIES

**Helena Gaspars-Wieloch**
Katedra Badań Operacyjnych i Ekonomii Matematycznej
Instytut Informatyki i Ekonomii Ilościowej
Uniwersytet Ekonomiczny w Poznaniu

| 1-criterion decision making under uncertainty (1-DMU) | Multi-criteria decision making under certainty (M-DMC) |
|---|---|
| The decision is made on the basis of **one objective function** | The decision is made on the basis of **at least two objective functions** |
| **At least one parameter** of the decision problem **is not known exactly** | **All the parameters** of the decision problem **are known** |
| **Pure/mixed strategies** | **Pure/mixed strategies** |

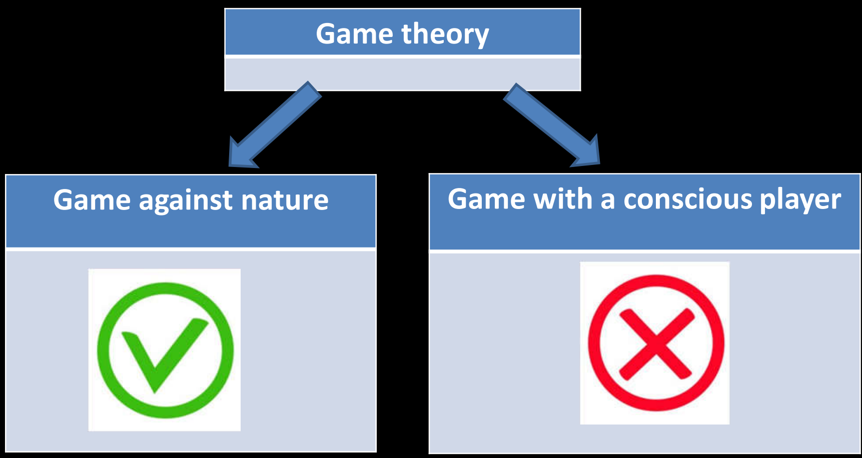
**Helena Gaspars-Wieloch**

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

| Uncertainty levels | |
|---|---|
| Uncertainty with **known probabilities** | → Scenario probabilities are given, the **expected value** may be computed |
| Uncertainty with **partially known probabilities** | → Scenario probabilities are **interval values** / scenarios are **ordered** according to their chance to occur |
| Uncertainty with **unknown probabilities** | → **Lack of scenario probabilities** |
| Uncertainty with **unknown scenarios** (ignorance) | → **Lack of payoff matrix** |

**Helena Gaspars-Wieloch**

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS




Helena Gaspars-Wieloch

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

1-criterion payoff matrix

| Scenarios \ Alternatives | $A_1$ | ... | $A_j$ | ... | $A_n$ |
|---|---|---|---|---|---|
| $S_1$ | $a_{1,1}$ | ... | $a_{1,j}$ | ... | $a_{1,n}$ |
| ⋮ | ⋮ | ⋱ | ⋮ | ⋱ | ⋮ |
| $S_i$ | $a_{i,1}$ | ... | $a_{i,j}$ | ... | $a_{i,n}$ |
| ⋮ | ⋮ | ⋱ | ⋮ | ⋱ | ⋮ |
| $S_m$ | $a_{m,1}$ | ... | $a_{m,j}$ | ... | $a_{m,n}$ |

1-DMU

Multi-criteria payoff matrix

| Criteria \ Alternatives | $A_1$ | ... | $A_j$ | ... | $A_n$ |
|---|---|---|---|---|---|
| $C_1$ | $b_{1,1}$ | ... | $b_{1,j}$ | ... | $b_{1,n}$ |
| ⋮ | ⋮ | ⋱ | ⋮ | ⋱ | ⋮ |
| $C_k$ | $b_{k,1}$ | ... | $b_{k,j}$ | ... | $b_{k,n}$ |
| ⋮ | ⋮ | ⋱ | ⋮ | ⋱ | ⋮ |
| $C_p$ | $b_{p,1}$ | ... | $b_{p,j}$ | ... | $b_{p,n}$ |

M-DMC

Helena Gaspars-Wieloch

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

## Papers

- On some analogies between one-criterion decision making under uncertainty and multi-criteria decision making under certainty ›
  Gaspars-Wieloch Helena, Economics and Business Review, 2021, vol. 7, nr 2, s.17-36. DOI:10.18559/ebr.2021.2.3
- A New Application for the Goal Programming - The Target Decision Rule for Uncertain Problems ›
  Gaspars-Wieloch Helena, Journal of Risk and Financial Management, 2020, vol. 13, nr 11, s.1-14. DOI:10.3390/jrfm13110280
- From Goal Programming for Continuous Multi-Criteria Optimization to the Target Decision Rule for Mixed Uncertain Problems ›
  Gaspars-Wieloch Helena, Entropy, 2022, vol. 24, nr 1, s.1-12, Numer artykułu:51. DOI:10.3390/e24010051
- From the interactive programming to a new decision rule for uncertain one-criterion problems ›
  Gaspars-Wieloch Helena, W: Proceedings SOR'21. The 16th International Symposium on Operations Research / Drobne Samo *[i in.]* (*red.*), 2021, Ljubljana, Slovenian Society Informatika, s.669-674, ISBN 978-961-6165-57-0
- Possible new applications of the interactive programming based on aspiration levels ›
  Gaspars-Wieloch Helena, W: 22nd International Conference on Quantitative Methods in Economics 2021: Book of abstracts, 2021, Warszawa, Wydawnictwo SGGW, s.20-20
- Possible new applications of the interactive programming based on aspiration levels - case of pure and mixed strategies ›
  Gaspars-Wieloch Helena, Central European Journal of Operations Research, 2022, nr first online, s.1-17. DOI:10.1007/s10100-022-00836-y

**Helena Gaspars-Wieloch**

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

## Interactive programming – interaction styles

→ binary pairwise comparison
→ vector comparison
→ precise local trade-off ratio
→ interval trade-off ratio
→ aspiration levels
→ index specification and value trade-off
→ comparative trafe-off ratio
→ pairwise comparison

**Helena Gaspars-Wieloch**

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

**Interactive programming for M-DMC** → **Interactive approach for 1-DMU (pure strategies)**

1. Define the set of options $A=\{A_1,\ldots,A_j,\ldots,A_n\}$ where $n$ is the number of alternatives.
2. Define the set of scenarios $S=\{S_1,\ldots,S_i,\ldots,S_m\}$ where $m$ is the number of scenarios.
3. Generate the payoff matrix (see Table 2, Section 3).
4. Choose the scenario with the highest subjective chance of occurrence ($l$=1).
5. Find set $A^1$, i.e. select the alternatives satisfying $AL_1$, i.e. the aspiration level declared for the aforementioned scenario by the DM.
6. Move to scenario $l$=2 (subsequent elements of the scenario sequence are established iteratively by the DM), and select alternatives fulfilling $AL_2$ from set $A^1$. Follow the same procedure for scenarios $l$=3,…,$m$-1.
7. Select from the current reduced set of alternatives (i.e. $A^{m-1}$) the option (or set of options) with the best payoff value within scenario $l$=$m$. That is the optimal pure strategy.

Helena Gaspars-Wieloch

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

**Interactive programming for 1-DMU – example (pure strategies)**

| *Scenarios* | *Alternatives* | | | | |
|---|---|---|---|---|---|
| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
| $S_1$ | 5000 | 2000 | 4000 | 1000 | 6000 |
| $S_2$ | 1500 | 1500 | 2000 | 1500 | 1000 |
| $S_3$ | 10000 | 8000 | 11000 | 4000 | 9000 |
| $S_4$ | 500 | 1000 | 0 | 1000 | 0 |

**Initial assumptions:** scenario order (S2, S1, S4, S3), *AL1*(S2)=1200

{A1, A2, A3, A4}

***AL2*(S1)=2000** {A1, A2, A3} → ***AL3*(S4)=400** {A1, A2} → ***AL4*(S3)=?** {A1}

Helena Gaspars-Wieloch

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

**Interactive programming for M-DMC** → **Interactive approach for 1-DMU (mixed strategies)**

1. Define the decision variables $x_1, \ldots, x_j, \ldots, x_n$.
2. Define the set of scenarios $S=\{S_1,\ldots,S_i,\ldots,S_m\}$ and the coefficients of their objective functions $f_1(x), \ldots, f_i(x), \ldots, f_m(x)$.
3. Formulate constraints connected with the decision problem:

$$\boldsymbol{x_j} \in \boldsymbol{SFS'} \qquad j=1,\ldots,n \qquad (14)$$

$$\boldsymbol{x_j} \geq \boldsymbol{0} \qquad j=1,\ldots,n \qquad (15)$$

$$\sum_{j=1}^{n} \boldsymbol{x_j} = \boldsymbol{1} \qquad (16)$$

where *SFS'* denotes the set of feasible solutions. Thus, equation (14) concerns all the conditions additionally formulated by the DM.
4. Establish iteratively the sequence of the analyzed scenarios $Seq=(S(1),\ldots,S(l),\ldots,S(t))$ where $t=p$.
5. Declare iteratively the aspiration levels: $AL(1), \ldots, AL(l), \ldots, AL(t\text{-}1)$.

**Helena Gaspars-Wieloch**

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

**Interactive programming for M-DMC** → **Interactive approach for 1-DMU (mixed strategies)**

6. Solve the optimization model (14)-(18):

$$\boldsymbol{f_{S(2)}(x) \rightarrow max/min} \qquad (17)$$

$$\boldsymbol{f_{S(1)}(x) \geq AL(1)} \qquad (18)$$

7. Solve the optimization model (14)-(16), (18) and (19)-(20):

$$\boldsymbol{f_{S(3)}(x) \rightarrow min/max} \qquad (19)$$

$$\boldsymbol{f_{S(2)}(x) \geq AL(2)} \qquad (20)$$

where $AL(2)$ is determined after solving the model (14)-(18) and finding the maximum and minimum possible value of function $f_{S(2)}(x)$.
8. Follow the same procedure for scenarios $S(3),\ldots,S(t\text{-}1)$ using equations (14)-(16) and (21)-(22).

$$\boldsymbol{f_{S(l)}(x) \rightarrow min/max} \qquad (21)$$

$$\boldsymbol{f_{S(1)}(x) \geq AL(1); \ldots; f_{S(l-1)}(x) \geq AL(l-1)} \qquad (22)$$

When solving the last optimization model (i.e. in the case where function $f_{S(t)}$ is optimized) there is no need to search both the maximum and minimum function value. It suffices to apply the direction of optimization desired by the DM. The result is the optimal mixed strategy.

**Helena Gaspars-Wieloch**

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

## IP for 1-DMU and mixed strategies - example

### Expected monthly rate of return in %

| *Scenarios* | *Alternatives* | | | | | | |
|---|---|---|---|---|---|---|---|
| | *A1* | *A2* | *A3* | *A4* | *A5* | *A6* | *A7* |
| *S1* | 6 | -2 | 14 | 14 | 7 | 20 | -12 |
| *S2* | 7 | -10 | 5 | 8 | 0 | 30 | -3 |
| *S3* | -10 | -9 | 4 | -14 | 12 | -40 | 40 |
| *S4* | -11 | 31 | 6 | -3 | -10 | -50 | 25 |

Assumptions:
- Max 20%
- x6<=x3
- (S3,S4,…….)
- AL(S3)=7%

I optimization model

$$-11x_1 + 31x_2 + 6x_3 - 3x_4 - 10x_5 - 50x_6 + 25x_7 \to max/min$$

$$-10x_1 - 9x_2 + 4x_3 - 14x_4 + 12x_5 - 40x_6 + 40x_7 \geq 7$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 1$$
$$0 \leq x_1, x_2, x_3, x_4, x_5, x_6, x_7 \leq 0.2$$
$$x_6 \leq x_3$$

Helena Gaspars-Wieloch

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

## IP for 1-DMU and mixed strategies – example

### Expected monthly rate of return in %

| *Scenarios* | *Alternatives* | | | | | | |
|---|---|---|---|---|---|---|---|
| | *A1* | *A2* | *A3* | *A4* | *A5* | *A6* | *A7* |
| *S1* | 6 | -2 | 14 | 14 | 7 | 20 | -12 |
| *S2* | 7 | -10 | 5 | 8 | 0 | 30 | -3 |
| *S3* | -10 | -9 | 4 | -14 | 12 | -40 | 40 |
| *S4* | -11 | 31 | 6 | -3 | -10 | -50 | 25 |

Assumptions:
- Max 20%
- x6<=x3
- (S3,S4,S2,.)
- AL(S4)=6%

II optimization model

$$7x_1 - 10x_2 + 5x_3 + 8x_4 + 0x_5 + 30x_6 - 3x_7 \to max/min$$

$$-10x_1 - 9x_2 + 4x_3 - 14x_4 + 12x_5 - 40x_6 + 40x_7 \geq 7$$
$$-11x_1 + 31x_2 + 6x_3 - 3x_4 - 10x_5 - 50x_6 + 25x_7 \geq 6$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 1$$
$$0 \leq x_1, x_2, x_3, x_4, x_5, x_6, x_7 \leq 0.2$$
$$x_6 \leq x_3$$

Helena Gaspars-Wieloch

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

## IP for 1-DMU and mixed strategies – example

### Expected monthly rate of return in %

| Scenarios | Alternatives | | | | | | |
|---|---|---|---|---|---|---|---|
| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ |
| $S_1$ | 6 | -2 | 14 | 14 | 7 | 20 | -12 |
| $S_2$ | 7 | -10 | 5 | 8 | 0 | 30 | -3 |
| $S_3$ | -10 | -9 | 4 | -14 | 12 | -40 | 40 |
| $S_4$ | -11 | 31 | 6 | -3 | -10 | -50 | 25 |

Assumptions:
- Max 20%
- x6<=x3
- (S3,S4,S2,S1)
- AL(S2)=1%

III optimization model

$$6x_1 - 2x_2 + 14x_3 + 14x_4 + 7x_5 + 20x_6 - 12x_7 \rightarrow max$$

$$-10x_1 - 9x_2 + 4x_3 - 14x_4 + 12x_5 - 40x_6 + 40x_7 \geq 7$$
$$-11x_1 + 31x_2 + 6x_3 - 3x_4 - 10x_5 - 50x_6 + 25x_7 \geq 6$$
$$7x_1 - 10x_2 + 5x_3 + 8x_4 + 0x_5 + 30x_6 - 3x_7 \geq 1$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 1$$
$$0 \leq x_1, x_2, x_3, x_4, x_5, x_6, x_7 \leq 0.2$$
$$x_6 \leq x_3$$

Helena Gaspars-Wieloch

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

## IP for 1-DMU and mixed strategies – example

### Expected monthly rate of return in %

| Scenarios | Alternatives | | | | | | |
|---|---|---|---|---|---|---|---|
| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ |
| $S_1$ | 6 | -2 | 14 | 14 | 7 | 20 | -12 |
| $S_2$ | 7 | -10 | 5 | 8 | 0 | 30 | -3 |
| $S_3$ | -10 | -9 | 4 | -14 | 12 | -40 | 40 |
| $S_4$ | -11 | 31 | 6 | -3 | -10 | -50 | 25 |

$x_1$=0.185,
$x_2$=0.132,
$x_3$=0.200,
$x_4$=0.083,
$x_5$=0.200,
$x_6$=0.000,
$x_7$=0.200.

S1 → 3.81%
S2 → 1.04%
S3 → 7.00%
S4 → 6.00%

Helena Gaspars-Wieloch

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

## IP for 1-DMU and mixed strategies – example 2

### Expected monthly rate of return in %

| Scenarios | Alternatives | | | | | | |
|---|---|---|---|---|---|---|---|
| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ |
| $S_1$ | 6 | -2 | 14 | 14 | 7 | 20 | -12 |
| $S_2$ | 7 | -10 | 5 | 8 | 0 | 30 | -3 |
| $S_3$ | -10 | -9 | 4 | -14 | 12 | -40 | 40 |
| $S_4$ | -11 | 31 | 6 | -3 | -10 | -50 | 25 |

Assumptions:
- Max 20%
- x6<=x3
- (S3,S4,…….)
- AL(S3)=7%
- SHORT SALE!

$$-11x_1 + 31x_2 + 6x_3 - 3x_4 - 10x_5 - 50x_6 + 25x_7 \rightarrow max/min$$

$$-10x_1 - 9x_2 + 4x_3 - 14x_4 + 12x_5 - 40x_6 + 40x_7 \geq 7$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 1$$
$$-0.2 \leq x_1, x_2, x_3, x_4, x_5, x_6, x_7 \leq 0.2$$
$$x_6 \leq x_3$$

I optimization model

Helena Gaspars-Wieloch

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

## IP for 1-DMU and mixed strategies – example 2

### Expected monthly rate of return in %

| Scenarios | Alternatives | | | | | | |
|---|---|---|---|---|---|---|---|
| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ |
| $S_1$ | 6 | -2 | 14 | 14 | 7 | 20 | -12 |
| $S_2$ | 7 | -10 | 5 | 8 | 0 | 30 | -3 |
| $S_3$ | -10 | -9 | 4 | -14 | 12 | -40 | 40 |
| $S_4$ | -11 | 31 | 6 | -3 | -10 | -50 | 25 |

Assumptions:
- Max 20%
- x6<=x3
- (S3,S4,S2,.)
- AL(S4)=6%
- SHORT SALE!

$$7x_1 - 10x_2 + 5x_3 + 8x_4 + 0x_5 + 30x_6 - 3x_7 \rightarrow max/min$$

$$-10x_1 - 9x_2 + 4x_3 - 14x_4 + 12x_5 - 40x_6 + 40x_7 \geq 7$$
$$-11x_1 + 31x_2 + 6x_3 - 3x_4 - 10x_5 - 50x_6 + 25x_7 \geq 6$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 1$$
$$-0.2 \leq x_1, x_2, x_3, x_4, x_5, x_6, x_7 \leq 0.2$$
$$x_6 \leq x_3$$

II optimization model

Helena Gaspars-Wieloch

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

## IP for 1-DMU and mixed strategies – example 2

### Expected monthly rate of return in %

| Scenarios | Alternatives | | | | | | |
|---|---|---|---|---|---|---|---|
| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ |
| $S_1$ | 6 | -2 | 14 | 14 | 7 | 20 | -12 |
| $S_2$ | 7 | -10 | 5 | 8 | 0 | 30 | -3 |
| $S_3$ | -10 | -9 | 4 | -14 | 12 | -40 | 40 |
| $S_4$ | -11 | 31 | 6 | -3 | -10 | -50 | 25 |

**Assumptions:**
- Max 20%
- x6<=x3
- (S3,S4,S2,S1)
- AL(S2)=1%
- SHORT SALE

$$6x_1 - 2x_2 + 14x_3 + 14x_4 + 7x_5 + 20x_6 - 12x_7 \rightarrow max$$

$$-10x_1 - 9x_2 + 4x_3 - 14x_4 + 12x_5 - 40x_6 + 40x_7 \geq 7$$
$$-11x_1 + 31x_2 + 6x_3 - 3x_4 - 10x_5 - 50x_6 + 25x_7 \geq 6$$
$$7x_1 - 10x_2 + 5x_3 + 8x_4 + 0x_5 + 30x_6 - 3x_7 \geq 1$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 1$$
$$-0.2 \leq x_1, x_2, x_3, x_4, x_5, x_6, x_7 \leq 0.2$$
$$x_6 \leq x_3$$

III optimization model

Helena Gaspars-Wieloch

POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS

## IP for 1-DMU and mixed strategies – example 2

### Expected monthly rate of return in %

| Scenarios | Alternatives | | | | | | |
|---|---|---|---|---|---|---|---|
| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ |
| $S_1$ | 6 | -2 | 14 | 14 | 7 | 20 | -12 |
| $S_2$ | 7 | -10 | 5 | 8 | 0 | 30 | -3 |
| $S_3$ | -10 | -9 | 4 | -14 | 12 | -40 | 40 |
| $S_4$ | -11 | 31 | 6 | -3 | -10 | -50 | 25 |

$x_1$=0.185,
$x_2$=0.132,
$x_3$=0.200,
$x_4$=0.083,
$x_5$=0.200,
$x_6$=0.000,
$x_7$=0.200.

S1 → 3.81%
S2 → 1.04%
S3 → 7.00%
S4 → 6.00%

Without short sale

$x_1$=0.199,
$x_2$=0.078,
$x_3$=0.200,
$x_4$=0.200,
$x_5$=0.200,
$x_6$=-0.055,
$x_7$=0.178.

S1 → 4.81%
S2 → 1.04%
S3 → 7.00%
S4 → 6.00%

With short sale

Helena Gaspars-Wieloch

## ADVANTAGES

1. No unitarization required
2. No need to use precise probabilities
3. More possibilities to take diverse DM’s preferences into account
4. Possibility to analyze scenarios iteratively
5. Possibilty to control all the values connected with a given option



**15.05.2023**

# POSSIBLE NEW APPLICATIONS OF THE INTERACTIVE PROGRAMMING BASED ON ASPIRATION LEVELS – CASE OF PURE AND MIXED STRATEGIES


**Helena Gaspars-Wieloch**
**Katedra Badań Operacyjnych i Ekonomii Matematycznej**
**Instytut Informatyki i Ekonomii Ilościowej**
**Uniwersytet Ekonomiczny w Poznaniu**